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Product adoption networks and their growth in a large mobile phone network

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Abstract—**To understand the diffusive spreading of a product in a telecom network, whether the product is a service, handset, or subscription, it can be very useful to study the structure of the underlying social network. By combining mobile traffic data and product adoption history from one of Telenor's markets, we can define and measure an *adoption network*—roughly, the social network of adopters. By studying the time evolution of adoption networks, we can observe how different products diffuses through the network, and measure potential social influence. This paper presents an empirical and comparative study of three adoption networks evolving over time in a large telecom network. We believe that the strongest spreading of adoption takes place in the dense core of the underlying network, and gives rise to a dominant largest connected component (LCC) in the adoption network, which we call "the social network monster". We believe that the size of the monster is a good indicator for whether or not a product is taking off. We show that the evolution of the LCC, and the size distribution of the other components, vary strongly with different products. The products studied in this article illustrate three distinct cases: that the social network monsters can grow or break down over time, or fail to occur at all. Some of the reasons a product takes off are intrinsic to the product; there are also aspects of the broader social context that can play in. Tentative explanations are offered for these phenomena. Also, we present two statistical tests which give an indication of the strength of the spreading over the social network. We find evidence that the spreading is dependent on the underlying social network, in particular for the early adopters.**

large-scale social network analysis; product spreading; telecommunications; time evolution;adoption networks; eigenvector centrality; mobile video telephony; iPhone; Doro; kappa-test; graph theory

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I. INTRODUCTION

This paper is motivated by the question of how people adopt new products and services, and what role the underlying social network structure plays in this process. The effect of the social network on product adoption and diffusion has been well documented in early market research, see e.g [9] for an overview of this research. Most of the early studies have suffered from limited network data availability, since social networks have traditionally been difficult to measure. Several theoretical network models have been developed. Some are less realistic due to the evolutionary nature and power law degree distributions [21][22]. Analyses of more realistic models can be found in [14] [23][24].

In recent years massive social network data have been made available to researchers through electronic phone logs [3][6][5][7] and online social network services [2][8]. These studies have confirmed that "the network matters" when customers decide to churn [4][6] and when purchase decisions are made [3][12]. Most of the existing research on product diffusion on networks has been focused on a single product, with a static snapshot of the social network. For an overview of analyses of large networks, the reader is referred to [13]-[20]. A few papers study the evolution of real-world networks [25]-[29]. In this paper we will present an empirical study of how the social network among adopters of telecom-products develops over time. In addition we will show how the product diffusion depends on the underlying social network.

We know that, for many products, a person's adoption probability increases with the number of that person's friends or contacts that have adopted the same product [3][6]. This can be interpreted as inter-personal or social influence, and can be measured empirically. These measurements do not typically say anything about the large-scale structure of the social network. In telecommunications it is possible to obtain detailed anonymized mobile traffic data for a large connected network of users. One can then use this telephony network as a proxy for the underlying social network. Furthermore, by combining telephone network data over time with the adoption history for a product of interest, it is possible to observe how different products spread over the social network.

Using anonymized datasets from one of Telenor's markets, we will show how two different handsets have spread over the social network. The cases being used in this study are the highly buzzed iPhone, and the less fancy, but user-friendly, Doro type handset, which is more common among elderly people.

Both handsets are tracked from their early introduction and followed for a period of two years. We also present the tracking of a *transactional* product, mobile video telephony, a potentially useful product which allows users to talk to each other, while simultaneously viewing one another (or one another's surroundings)—given certain technological preconditions.

We start by introducing the *adoption network*, a construction which is readily visualized and which gives insight into the spreading of a product or service. Our figures will include many visualizations, which, we believe, are useful in understanding the product diffusion process on the underlying social network.

II. The adoption network

We will in this section define what we mean by the adoption network, followed by an empirical example.

A. *Definition of adoption network*

We define an adoption network as follows. Given a measured telephony network, the node set of the adoption network is the set of subscribers that have adopted a given product, and the links are the communication links belonging to this subset.

Mathematically, an adoption network is thus a subgraph of the whole mobile communication network $C = (c_{ij})$, where C represents (most generally) a weighted, directed, and possibly disconnected graph.

The mobile communication matrix C places a link between each pair of communicating subscribers, so that each nonzero element c_{ij} represents communication. The communication can be based for example on a weighted sum of SMS and voice duration (in which case we call these weighted links *W-links*), or other transactional data like video telephony traffic. All the results in this paper depend only on whether the communication link exists or not, without consideration to weight or direction. We consider traffic between Telenor subscribers in one market, which implies that the matrix will be $n \times n$ large, where n is the size of the customer base (several million subscribers).

The adoption network is then simply the subgraph of C formed by including only the adopting nodes and their common links. As we will see, for a *transactional* product (video telephony), there are two distinct useful choices for the communication links to be used in defining the adoption graph: (i) the standard (voice + SMS) links, or (ii) the links representing the use of the transactional service.

B. *Introducing the "social network monster" by example*

Figure 1 shows the empirical iPhone adoption network from Q4 2007. (This was measured before the iPhone had been introduced into the Telenor net; hence these users have presumably bought their iPhones in the US and "cracked" them for use on the Telenor net). The data show that 42% of the iPhone users communicated with at least one other iPhone-user, which speaks to the social nature of technology consumption, while 58% did not have any iPhone contacts. We call the latter *isolates*. We do not include isolates in any of our visualizations of adoption networks, but do include them in all results counting number of users. We also study the connected components of the adoption network, where the connected components are subgraphs in which any two nodes are connected to each other by paths. Using this convention, we find for example that the largest connected component (LCC)

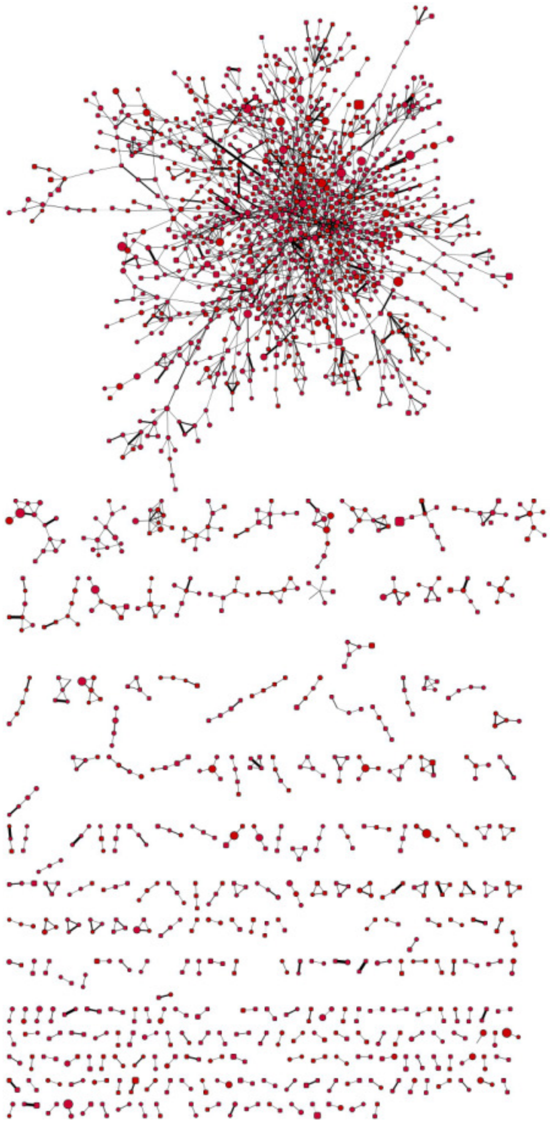

Figure 1. iPhone Q4 2007 adoption network. One node represents one subscriber. Node size represents downloaded internet volume. Link width represents a weighted sum of SMS+voice. Isolates—adopters who are not connected to other adopters—are not shown in the picture.

in the adoption network of Figure 1 includes 24.7% of the total number of adopters (while representing over half of the nodes visible in Figure 1). When the LCC in the adoption network is much bigger than all other connected components, and also represents a large fraction of all adopters, we will call the LCC a "social network monster". We note that this is not a precise definition; but we find that such monsters are typically found in adoption networks, and hence believe that the concept is useful.

III. TIME EVOLUTION OF ADOPTION NETWORKS

By studying the time evolution of an adoption network, we can get some insight into how the product which defines the adoption network is diffusing over the underlying social network. In particular we will often focus on the time evolution of the LCC of the adoption network – which may or may not form a social network monster. We recall from Figure 1 that the other components are often rather small compared to the LCC. Hence we argue that studying the evolution of the LCC itself gives useful insight into the strength of the network spreading mechanisms in operation. It also gives insight into the broader context of adoption. As described in [8], two friends adopting together does not necessarily imply social influence – there might also be external factors that control the adoption. In this paper we will not try to separate the 'influence-effects' from external effects such as network homophily. Instead, when we observe a tendency that people who talk together also adopt together, we will use the term 'social spreading'—without making any implicit claim as to the underlying mechanism.

A. *The iPhone case*

The iPhone 2G was officially released in the US in late Q2 2007 followed by 3G in early Q3 2008 and 3GS late Q2 2009. It was released on the Telenor net in 2009. Despite the existence of various models, we have chosen to look at the iPhone as one distinct product, since (as we will see) the older

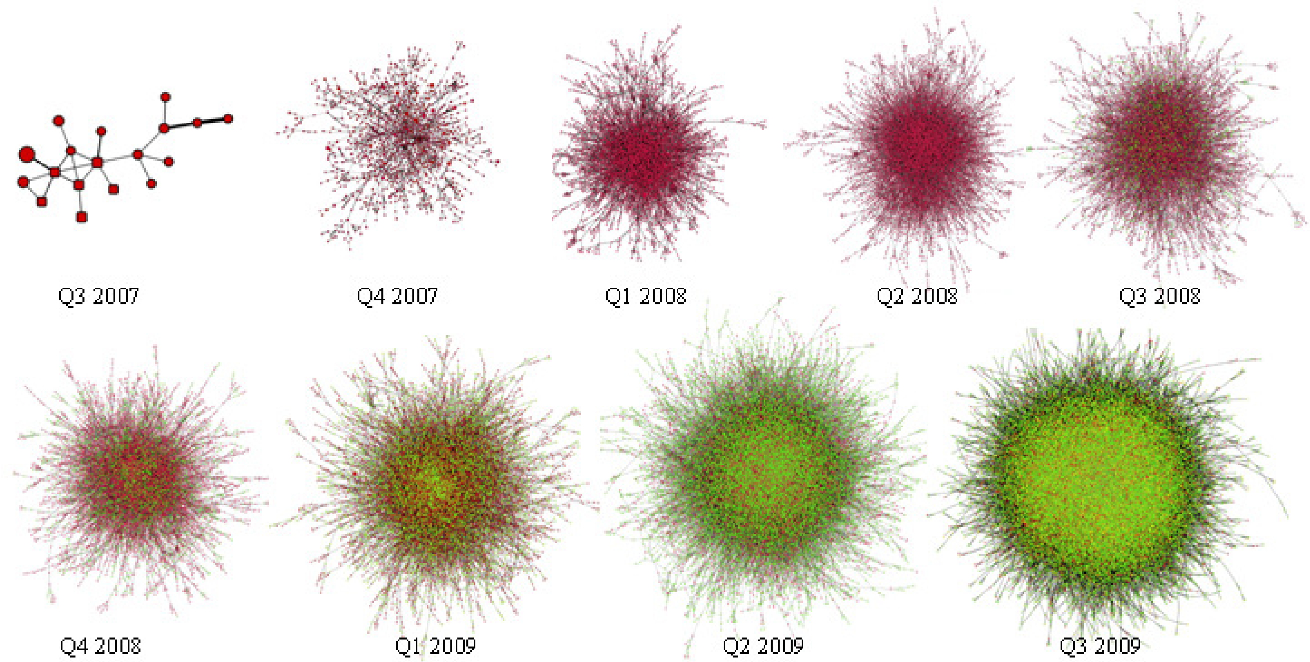

Figure 2. Time evolution of the iPhone adoption network. One node represents one subscriber. Node color: represents iPhone model: red=2G, green=iPhone 3G, yellow=3GS. Node size, link width, and node shape (attributes which are visible in Q3 2007) represent, respectively, internet volume, weighted sum of SMS and voice traffic, and subscription type. Round node shape represents business users, while square represents consumers.

models are naturally substituted in our network. Figure 2 shows the development of the iPhone monster in one particular market. We observe how the 2G phone is gradually substituted

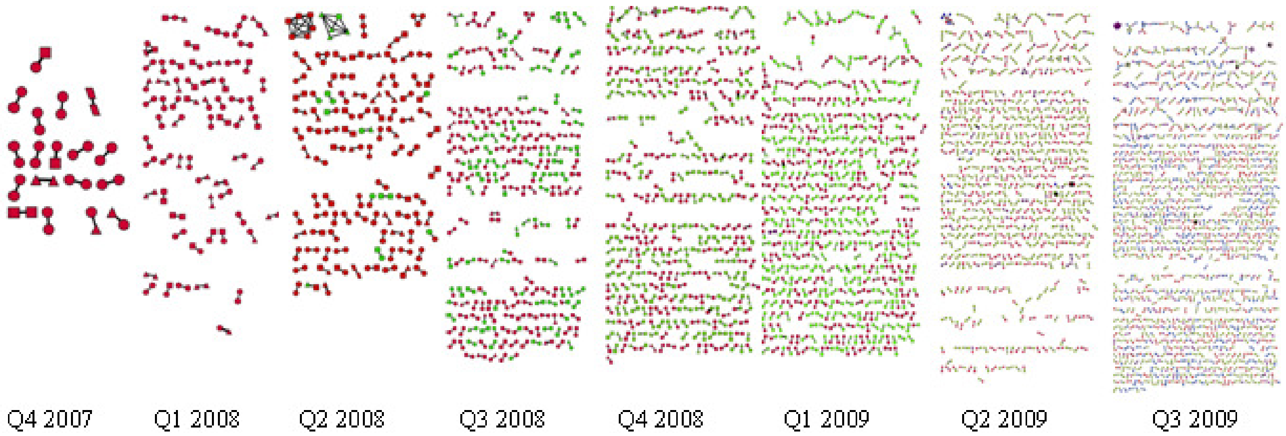

Figure 3. Time evolution of Doro adoption network. One node represent one subscriber. Node color represents Doro Model: red=HandleEasy 326,328, green=HandleEasy 330, blue=PhoneEasy 410, Purple=Other Doro models. Node shape represents age of user: A circle means that user is older than 70 year. Link width represents weighted sum of SMS and Voice traffic.

by 3G (red to green), followed by 3GS in Q3 2009 (yellow nodes). The 2G model falls from 100% to 10% with respect to all the iPhone subscribers in the adoption network. In Q1 2009 we observe the same amount of 2G as 3G models.

We show only the LCC in Figure 2 because the other components are visually very much like those seen in Figure 1; the main change over time is that the non-LCC components increase greatly in number, but not in size. That is, essentially all significant growth in component size occurs (in the iPhone case) in the LCC. We regard this growth as a sign that the iPhone is spreading strongly ("taking off") over the social network. It is worth noting that there is a significant marketing "buzz" and external social pressure associated with the iPhone that is perhaps unique. We will offer in later Sections other kinds of measurements which support this conclusion.

B. *The DORO case*

The next example is the Doro. As with the iPhone, there are several different models that are considered collectively. It is a handset which is easy to use, and mainly targeted towards elderly people [10]. Since Doro has a relatively low number of non-isolated users in all quarters studied, we present in Figure 3 visualizations of the whole adoption network (minus isolates) over the entire time period, from introduction (in Q4 2007) to Q3 2009. Figure 3 shows that most Doro users that are not isolates appear in pairs in the adoption network. The social network monster never appears—the contrast with the iPhone case is striking. We believe that the kind of adoption network evolution seen in Figure 3 is indicative of a product where "buzz" effects—social influence in the spreading of adoption—are weak or absent, whereas what we see in Figure 2 indicates strong buzz effects. It is possible to argue that the adoption of the Doro is more of an individual choice, or perhaps even the choice of the user's children who wish to be in contact with their elderly parents. We note finally that the tiny "monster" (LCC) seen in Q3 2009 of Figure 3 consists entirely of enterprise subscribers. Hence we speculate that these users are not the elderly of the target segment, but rather users with some other interest in the product.

C. *Mobile Video Telephony case*

Compared to iPhone and Doro, video telephony has no value for an isolated user; thus users will always appear in pairs. A similar (pairs-only) constraint may be seen in [1], where the connections are based on romantic relations. In the video telephony case we actually have two distinct link sets which may be used to define an adoption network: W-links (voice + SMS), and video links. Thus for mobile video telephony we create and study two distinct adoption networks:

- The video-link set gives rise to the video adoption network (VAN)

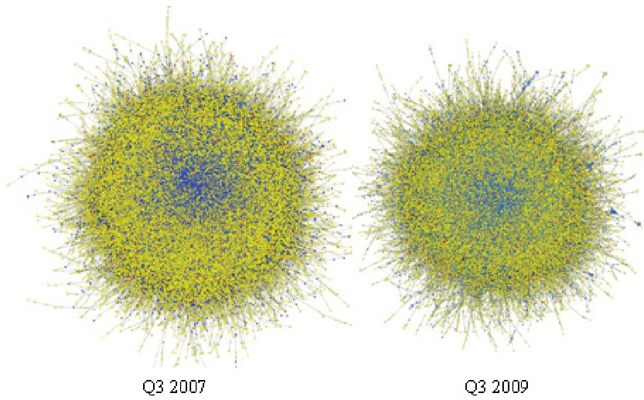

Figure 4. Time evolution of Mobile Video Telephony adoption network (WVAN—where the social links include all communication). Only two quarters are shown due to the fairly stable LCC. Blue node color represents enterprise subscribers, while yellow represents private subscribers.

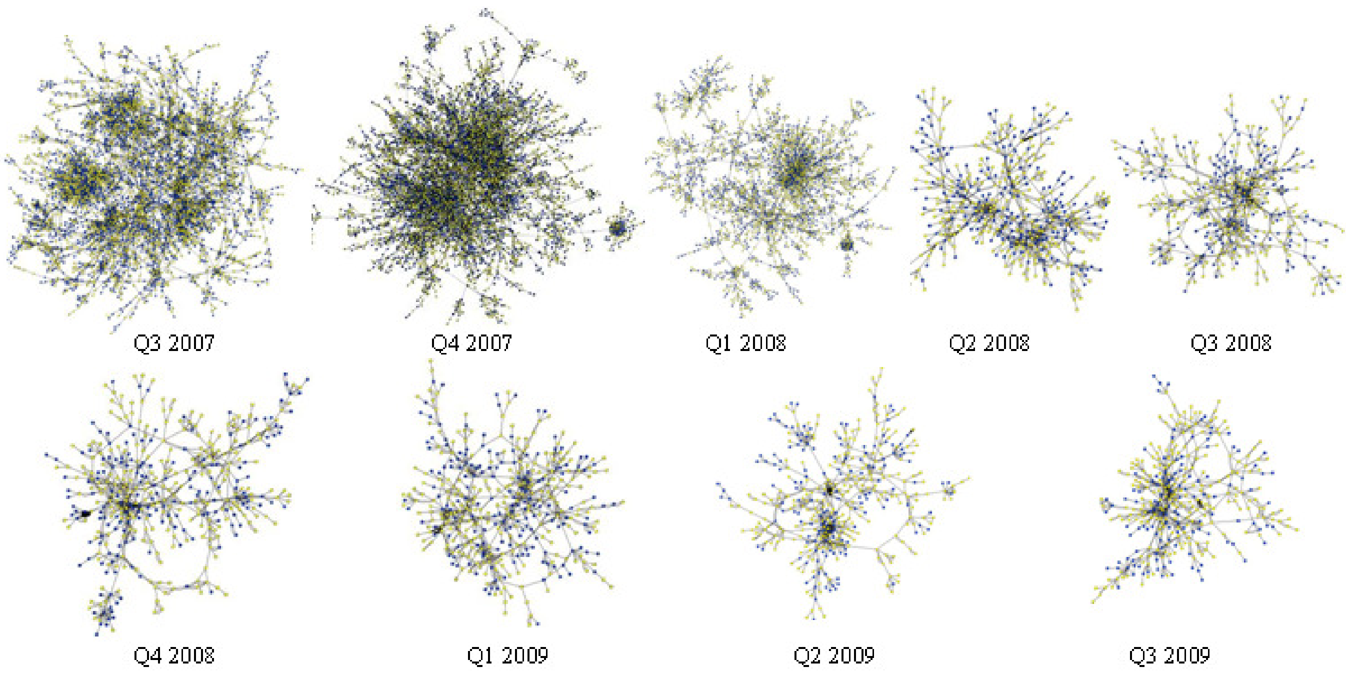

Figure 5. Time evolution of the Mobile Video Telephony adoption network. Links are real video links, where width represents duration of video conversations. Enterprise adopters have blue node color, while consumer adopters are tagged yellow.

- The W-linkset (voice+SMS) gives rise to the W-Video adoption network (WVAN).

We find that these two networks for video telephony have quite different behavior. We consider first the WVAN. This network connects users who (i) have a communication link (voice and/or SMS) and (ii) both use video telephony—not necessarily with each other. For the WVAN we find consistently a large social monster, much like the one seen in Figure 2 that starts to form. However, differently from Figure 2, the monster in the WVAN actually diminishes in size over time—both in absolute number of users, and in the percentage of users in the LCC. Figure 4 illustrates this by showing two WVAN video-monsters which are two years apart.

We gain even more insight by looking at the time evolution of the VAN. Figure 5 shows the time evolution of the VAN-LCC. We see growth in the monster from Q3 2007 to Q4 2007, followed by a rather dramatic breaking down of the LCC after that time. Hence we see indications that the service itself had the potential to form a real social monster and take off, but some change in the service and user conditions killed that takeoff – in this case, we have found that a new pricing model was introduced.

D. Comparison of the social network monsters over time

Figure 6 sums up much of what we have seen in the visualizations of the last subsections. The figure shows the fraction of adopters in various components of the adoption network. Subscribers in the blue area are adopters which have no connection to other adopters. These users (termed isolates here, and referred to as singletons in [2]) have not been visible in our visualizations. The users in the green area correspond to the adopters in the social network monster (there is in every case only one component with >1000 users).

We first consider Figure 6(a), the figure describing iPhones. Here we see that the growth of the monster (green), as a percentage of the total number of users, has not been monotonic. The monster has however grown monotonically in the absolute number of users—see again Figure 2. We conclude from this that the number of isolated subscribers grew more rapidly than did the core. This implies that some change in the offering has induced a large growth in the number of new users in this period (Q2 2008—Q3 2008). One candidate explanation is the appearance of 3G handsets in this time period. Another likely explanation for many new users is the fact that "legitimate" iPhones were first available on the Telenor net at this time.

Figure 6(b) (Doro) simply confirms the picture seen in Figure 3: no monster, essentially no large LCCs. At the same time we see an enormous dominance of isolates. This is consistent with the hypothesis that Doro users are elderly (which we can confirm), and that they speak mostly with other generations, ie, non-Doro users. Again, it suggests that the adoption of this phone is not based on network influence, but on more ego-based considerations.

Figure 6(c) shows the VAN, while 6(d) shows the WVAN for the video product. Again we confirm the qualitative picture obtained from Figs. 4 and 5: the WVAN-monster decays slowly, while the VAN-monster collapses. In the case of the video service, the collapse corresponds to the initiation of payment for the system. Using the lingo of the iPhone example, this would be the same as turning of the "buzz". We also see in Figure 6(c) a dominance of two-node components—not surprising for a transactional service—and a complete absence of isolates. The latter result, while not surprising, is not in fact guaranteed (for WVAN) by our definitions: we will see two

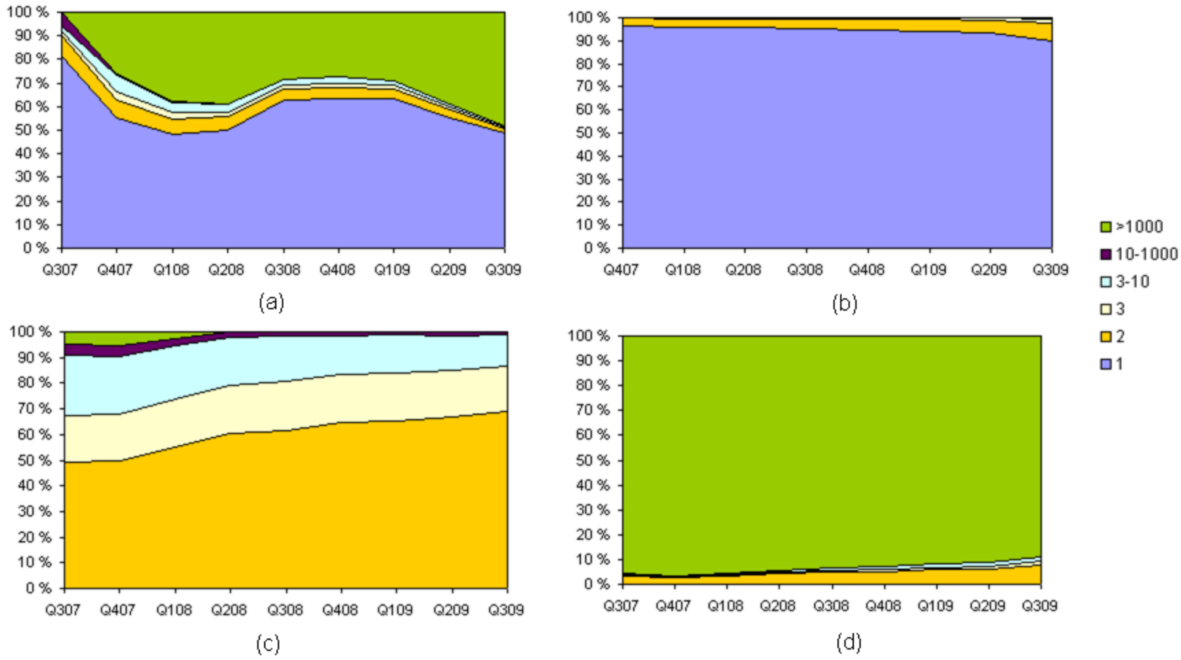

Figure 6. Fraction of subscribers in components of various sizes in the adoption networks for: (a) iPhone (b) Doro (c) Video (video-links) (d) Video (W-links)

isolates in WVAN every time two subscribers use video transactions, but have no other (W) communication, and have no friends using video transactions. We see that this simply does not happen—primarily because every pair that uses video also uses voice, SMS, or both.

We offer some quantitative details illustrating the dynamics seen in Figs. 6(c) and 6(d). We observe that 95.8% of users are in the core of WVAN in Q3 2007, while only 5.7% are in the VAN core. Two years later, the corresponding numbers are 88.7% for WVAN and 0.76% for VAN.

IV. CENTRALITY OF ADOPTERS

We have seen how the iPhone and video adopters form a giant monster with respect to the W-links, while the Doro adopters do not. Motivated by this, we calculate the social centrality for all the adopters in each group, comparing it to the centrality of the whole customer base. Our expectation is that the involvement of highly central users is essential to the development of a large monster. To measure centrality we use the well-known eigenvector centrality (EVC). We believe high EVC will be strongly correlated with presence in the social monster, as it is already known to be correlated with strong spreading [11].

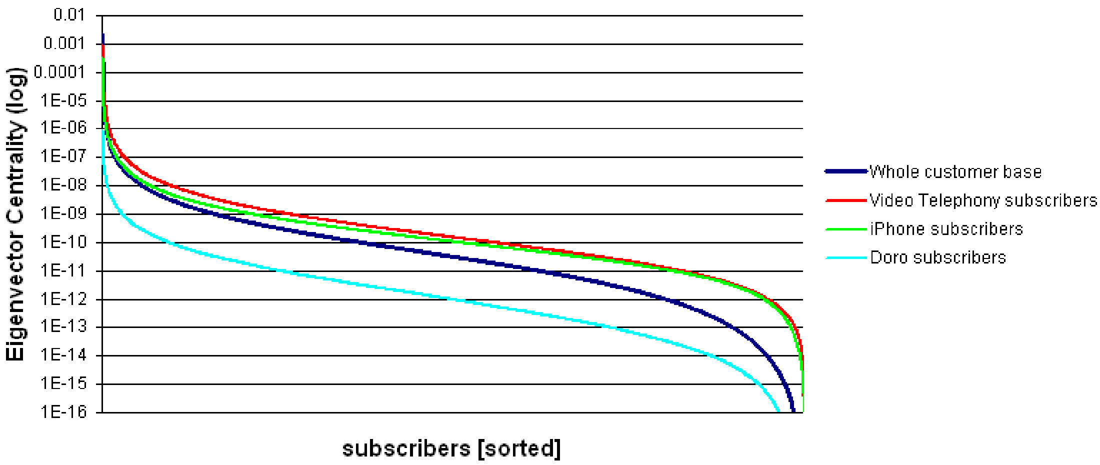

Figure 7. Eigenvector centrality distributions for the whole customer base, Doro users, iPhone users, and video telephony users. Distributions are from Q3 2009. For ease of comparison, the x-scale is normalized so as to run from 0 to 100% for all displayed distributions.

Figure 7 shows the EVC distributions for all the adopters. We see that video users are the most central, with iPhone just behind. We also see that the Doro adopters are rather peripheral socially, with their distribution falling well below that for the entire customer base. This supports our expectation that people in the giant monsters tend to be more central than the rest of the customer base. We believe that one may find, among these customers, the influential early adopters—those that adopt new products and services fairly early, and stimulate (or perhaps demand) others to do the same.

V. KAPPA-TEST

In all our results so far we see indirect evidence for social spreading effects (or their apparent absence, in the Doro case). As another test for social spreading, we introduce a simple statistical test, the kappa-test or κ-test.

A. *Definition of the κ-test*

We consider again the entire social network (as proxied by our communication graph) and define two types of links:

- A-links: links where neither, or only one, of the two connected nodes have adopted the product.
- B-links: links where the two connected nodes have both adopted the product.

We regard B-links to be the links which can indicate (but not confirm) social influence. We also recognize however that B-links can arise by other mechanisms, and even by chance. In order to evaluate the significance of the B-links that we observe in the empirical adoption data, then, we compare the empirical number of B-links (call it $n_{\mathrm{B,emp}}$) with the number found by distributing at random the same number of adopters over the same social network, and then counting the resulting number of B-links $n_{\mathrm{B,rand}}$.

We then define $\kappa \equiv n_{\mathrm{B,emp}} / n_{\mathrm{B,rand}}$. Clearly, if κ is significantly larger than 1, we have strong evidence for social spreading effects. More precisely, κ>1 implies that people who communicate with each other tend to adopt together.

Figure 8 illustrates what happens when we scatter the iPhone adopters in Q1 2009 randomly over the empirical social network. The monster is still there, but it is smaller (by more than a factor 3) than the empirical monster seen in Figure 2. Comparing the corresponding whole adoption networks of Figs. 2 and 8 by using the κ-test, we find that there are over twice as many links in the empirical adoption network compared to the random reference model—that is, κ is 2.18. We take this to be evidence that social spreading has occurred—more precisely, that people who talk together adopt together much more often than chance would predict.

Figure 8 illustrates an important point which gives insight into both monsters and social network structure. The point is that monsters arise even in the complete absence of social spreading effects. The monster seen in Figure 8 is thus telling us something about the structure of the social network itself—that it has a "dense core" in which a dominating LCC arises even in the case of random adoption. At the same time, this dense core must include the set of users who give rise to the empirical monsters that we observe. The empirical LCC is simply larger than the random one (for the same number of adopters), due to social spreading (arising from mechanisms such as social influence or homophily).

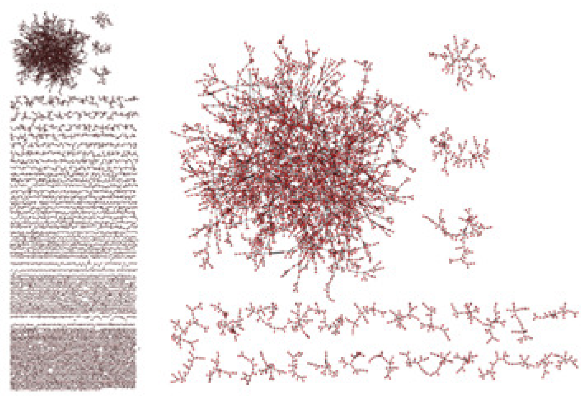

Figure 8. a) iPhone network from random reference model, Q1 2009. b) LCC zoomed in

Figure 9 shows the evolution of κ over time, both for the iPhone and for Doro. We notice that κ is very large in the early stages of product adoption (eg, around 28.7 for the iPhone in

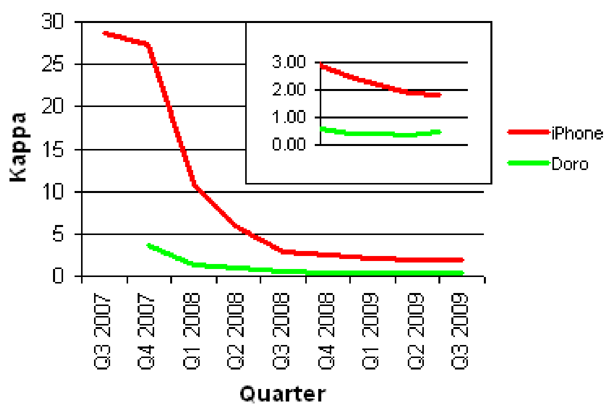

Figure 9. Kappa for Doro and iPhone

Q3 2007). We find this to be typical: the first adopters are not randomly distributed, but rather tend to lie in a few small connected social groups. The large value for κ tells us that this observed distribution of the early adopters on a social network is extremely unlikely to have occurred by chance.

We notice also that κ is consistently less than 1 for Doro, after the early phase of adoption. While we argue that κ>1 is evidence for social spreading effects, we do not believe that κ<1 proves that such effects are not occurring. What κ<1 does say is that adopting friends are found less often than a random model would predict. Our explanation for this is that the random model hits the dense core more often than the actual empirical adopters do. In other words, the empirical adopters are socially peripheral. This idea is in agreement with the EVC distribution seen in Fig 6.

Finally we note that we have not performed our κ test for the video adoption network. The reason is that (as discussed in Section III) the transactional nature of the video service constrains both VAN (exactly) and WVAN (empirically) such that there are no isolates. This constraint is not captured by the random reference model of the κ test.

VI. CORRELATED ADOPTION PROBABILITY

Our final test for social spreading effects is to measure the probability p_k that a subscriber has adopted a product, given that k of the subscriber's friends have adopted the product. This conditional probability does not indicate causation, because it makes no reference to time order—it simply measures (again) how strongly those that communicate together tend to adopt together. We measure p_k simply by first finding all subscribers with k adopting friends, and then finding the fraction of these that have themselves adopted.

Figure 10 shows p_k vs k for the three products, for $0 \le k \le 3$. For higher k, the Doro data are too dominated by noise (a low n) to be useful. (The results for iPhone and video have better adoption profiles, and so are meaningful at least out to k=10; but their qualitative behavior—monotonic increase, at roughly constant slope, with increasing k—is like that seen in Figure 10.) Figure 10 supports our claim that there are some social spreading effects operating on Doro adoption—since we see a

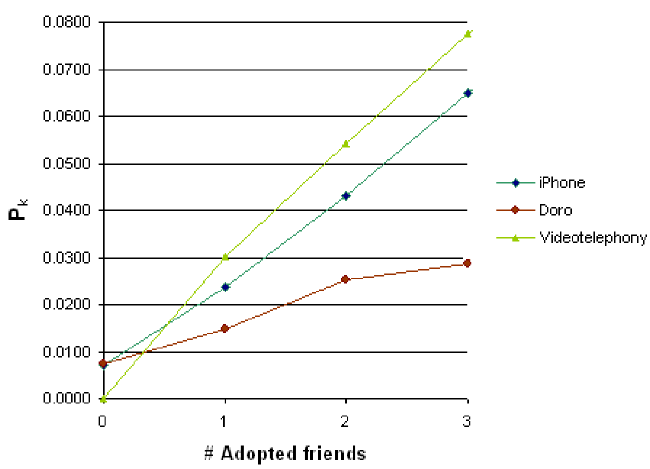

Figure 10. Adoption probability p_k vs the number k of adopting friends, for three products. In each case we see a monotonic growth of p_k with k—indicating that some kind of social spreading is occurring.

steady increase of p_k with k. Such effects are not visible from our κ test, for reasons given above; yet we see for example that, if we know that a subscriber has one friend using a Doro phone, that subscriber's probability of using one him- or herself is roughly *twice* the adoption probability for a subscriber with no adopting friends.

The iPhone and video p_k results lie, not surprisingly, considerably higher than the Doro curve. This is consistent with our claim that social spreading effects are much stronger for these products. For example, knowing that a person has one friend using an iPhone roughly triples the probability (compared to someone with no adopting friends) that this person also uses an iPhone. This comparison is however not possible to make for the video service, because the probability of using video telephony and having no (W-)friends who use video telephony is (as discussed above) empirically zero. Otherwise the video results are qualitatively the same as the iPhone results.

VII. SUMMARY AND FUTURE WORK

All of our results support a simple and fairly consistent interpretation:

- The iPhone has very strong social spreading, and has truly taken off
- The Doro handsets have only very weak social spreading. This device will probably never take off in the same sense as the iPhone.
- Video telephony use also has strong social effects, and started spreading very strongly; however its early takeoff was stopped by an external factor—here, a new price model.

Standard whole-network measures, such as total number of users, or total traffic over time, can also give useful information on these same questions. We believe however that our measurement methods give new and useful insight into how and why these services have performed so differently.

We have not performed a κ-test for the VAN, because the VAN has a fundamental constraint—that adoption occurs only in pairs—that our simple κ-test does not capture. We are now looking at a a generalized version of the κ-test, in which the unit of adoption is a pair (a link), and B-link counting is replaced by counting 'B-link pairs' (connected triples in VAN). We hope to report results from this kind of test (which should be useful for any transactional graph with the pair constraint) soon.

Also, we suspect that social spreading effects for the Doro handsets may be more visible at the two-hop level. A typical scenario might be an adult child would buy this type of phone for one or more of their elderly parents when, for example, use of a more traditional mobile phone becomes difficult because of the size of the display or keypad, or the complexity of the device. In such a scenario, it may well be there is little or no direct communication among the adopters, but a strong two-hop connection via the younger generation. We plan to test this idea in the near future.

ACKNOWLEDGMENT

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